Risk Allocation for Design Optimization with Unidentified Statistical Distributions

SciTech AIAA Non-Deterministic Approaches

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Reliability Based Design Optimization (RBDO) and Distributions

- The RBDO community often assumes you can identify statistical distributions
- **It is difficult to identify statistical distributions in practice**
- Regulators (e.g. FAA) tell you what to do when you can not identify the statistical distribution

Perhaps the regulators are more statistically savvy!

Obtaining conservative failure allowables

- You have performed a handful of tests on a material
- What failure strength do you use? (failure allowable)
- Deal with epistemic and aleatory uncertainty
- How to conservatively estimate the failure strength
- Various tolerance interval methods

Outline today

1. Conservative estimation of failure strength
2. Non-parametric and Hanson Koopmans tolerance intervals
3. Simple risk allocation RBDO for UAV redesign

Conservative estimate of failure strength

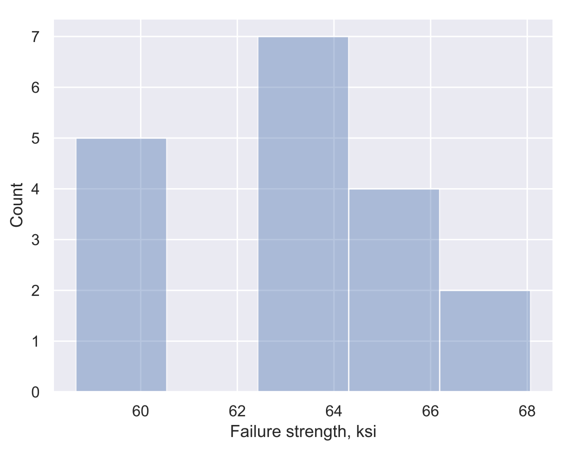

Figure 1: Histogram of 18 tension tests on composite material.

One-sided tolerance interval to estimate allowable strength

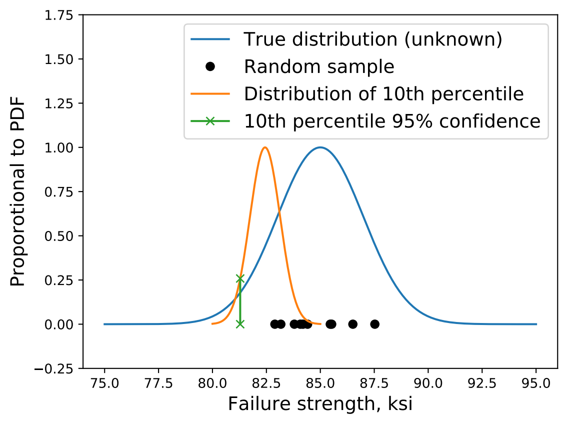

Figure 2: Estimating the 10th percentile to 95% confidence.

One-sided tolerance intervals for Aerospace

- 1st percentile to 95% confidence (A-basis): used in non-redundant structures
- 10th percentile to 95% confidence (B-basis): used in redundant structures
- FAA regulations on how to calculate failure strength allowables
- If distribution is known, easy to calculate!

Issues with real test data

It's difficult in practice to identify a known distribution

- You may have too much data
    - Tiny deviations from a distribution are enough to reject that samples come from that distribution
- You may have too little data

I'm not sure where this sweet spot exists...

Example with too little data

Consider this sample of 10 (from standard Normal distribution)

x = [0.98, -0.7, 0.95, -1.67, -1.4, 0.73, -0.2, 1.76, 1.18, 1.62]

Table 1: KS-Test 95% confidence: reject distribution if P-value < 0.05

Distribution	P-value	Reject?
Normal	0.57	False
Lognormal	0.57	False
Weibull	0.92	False
Gamma	0.52	False
Student's t	0.57	False

The random sample could have come from just about any distribution!

Non-parametric order statistic tolerance interval

Order a random sample $\boldsymbol{x}$ as

$$x_1 \leq x_2 \leq \cdots \leq x_n \tag{1}$$

then the non-parametric tolerance interval for P and γ confidence is expressed as

$$x_i \tag{2}$$

where i is determined from P and γ

Solves the tolerance interval problem with large samples!

However, it doesn't work well with small data.

Non-parametric failure strength on previous 18 samples

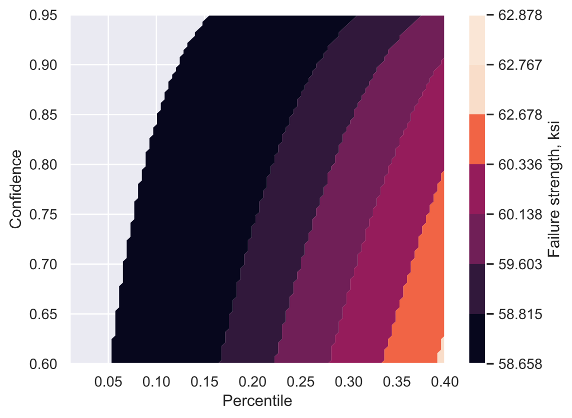

Figure 3: With just 18 samples, the non-parametric tolerance interval is limited to certain percentile confidence levels.

Hanson-Koopmans extends non-parametric for small samples

The tolerance interval is defined as

$$x_j - b(x_j - x_i) \tag{3}$$

where b is solved depending on i, j, P, γ, N

- b has been difficult to solve for (until today...)
- Sporadic use in FAA, SAE, mil-spec...
- Hanson-Koopmans assumes the true distribution is in the Log-Concave CDF class
- Non-parametric assumes the true distribution is continuous

Hanson-Koopmans failure strength of previous 18 samples

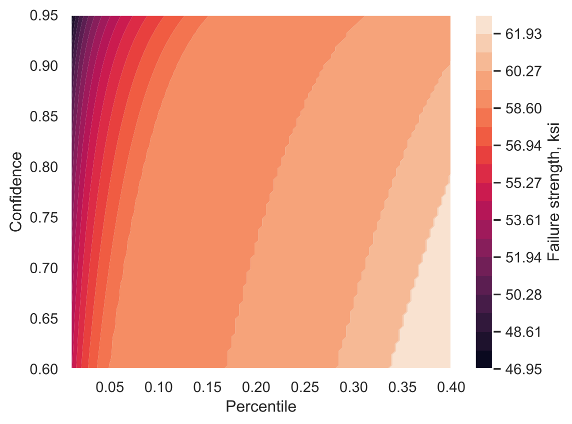

Figure 4: With just 18 samples, the Hanson-Koopmans tolerance interval can be calculated for any percentile and confidence.

My tiny Python library for tolerance intervals

https://github.com/cjekel/tolerance_interval_py

- Calculate one-sided tolerance intervals for Normal, Lognormal, Non-parametric, and Hanson-Koopmans methods
- Calculate Hanson-Koopmans b for any j, N, P, γ

```
import numpy as np
import toleranceinterval as ti
x = np.random.random(100)  # random sample of n=100
# estimate the 10th percentile to 95% confidence
bound = ti.oneside.hanson_koopmans(x, 0.1, 0.95)
```

So what is the Log-Concave CDF class?

- Fairly common statistical distribution class
- Includes all: Normal, Exponential, Gumbel, Laplace, Logistic, Rayleigh, Maxwell, Uniform, Lognormal, and Pareto distributions
- Also includes subsets of other distributions
- Composite material handbook: composite failure strength generally follows the Log-Concave CDF class

Visual example of what is Log-Concave CDF

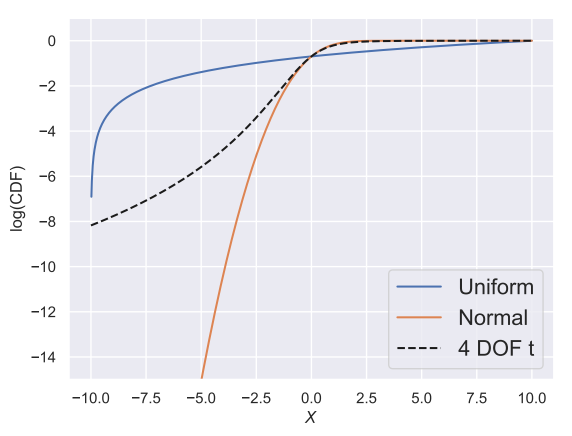

Figure 5: The log of the CDF for the Uniform and Normal distributions is concave, while Student's t-distribution in convex.

Risk allocation

Risk allocation

Individual components have different probabilities of failure.

Some potential advantages of risk allocation:

- Lower weight for same system probability of failure
- Lower system probability of failure for same weight

Application to risk allocation of UAV

- Apply Hanson Koopmans methods to do redesign a UAV
- Initial UAV design assumed components have equal safety margins (similar to FAA regulations)
- Redesign the wing and horizontal tail to have difference probabilities of failure

Initial design of UAV

- Takeoff weight 15 lbs
- Wing weight 1.35 lbs
- Horizontal tail weight 0.3 lbs
- Wingspan 9 ft
- Both wing and tail are fully stressed
- Stress allowable 1st percentile to 95% confidence from Hanson Koopmans on the 18 tests

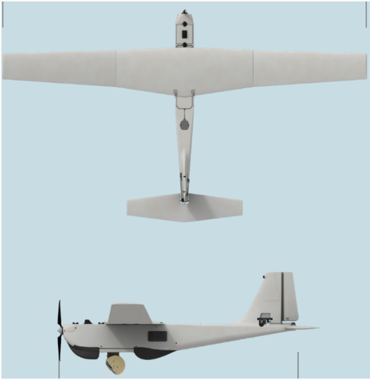

Figure 6: Image of Puma 3 AE by AeroVironment® inspired these design specs.

Assumptions to setup the risk allocation

The wing and tail failures are assumed to be independent, thus the system probability of failure is

$$P_f = 1 - (1 - P_w)(1 - P_t) \tag{4}$$

where P_w and P_t are the failure allowables.

The component weight can be assumed to be inversely proportional to change of failure allowable

$$\frac{\sigma_i}{\sigma_n} = \frac{W_n}{W_i} \tag{5}$$

from changing the skin thickness.

Simple risk allocation RBDO

- Minimize the weight of the wing and horizontal tail
- By changing the allowable failure strength for each component
- Such that the system has the same probability of failure

$$\min W = w_w(p_w, \gamma_w) + w_t(p_t, \gamma_t) \tag{6}$$

$$\text{such that: } P_f \leq 0.02 \tag{7}$$

$$\gamma_w = \gamma_t = 0.95 \tag{8}$$

Results when using the Hanson-Koopmans method

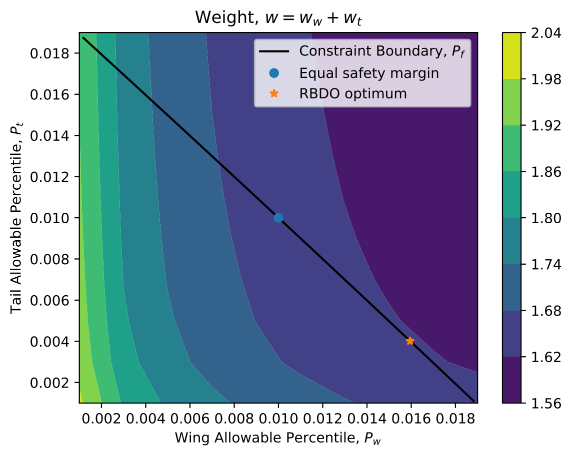

Figure 7: Contour plot of the objective function when using the Hanson-Koopmans method.

Results along the constrain boundary

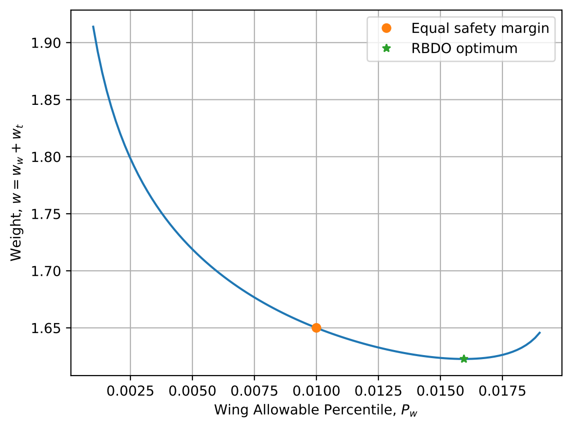

Figure 8: Line plot of the objective function along the constrain boundary when using the Hanson-Koopmans method.

Results comparing initial design to risk allocation

Table 2: Comparison of the UAV specifications from the previous equal safety margin design and the new RBDO optimal design.

	Equal safety margin Hanson-Koopmans	RBDO Hanson-Koopmans	RBDO Normal
P_f	0.020	0.020	0.020
P_w	0.010	0.016	0.017
P_t	0.010	0.004	0.003
w_w, lb	1.35	1.30	1.16
w_t, lb	0.30	0.32	0.27
w, lb	1.65	1.62	1.43

Lessons from risk allocation

- Lighter UAV for the same probability of failure using risk allocation
- Risk allocation between Hanson Koopmans and Normal distribution resulted in similar component failure probabilities
- Hanson Koopmans UAV was $\approx$ 15% heavier than using a Normal distribution

Overall conclusion

- RBDOs often assume that it is possible to identify statistical distributions
- It is difficult to identify statistical distributions
- Regulation (e.g. FAA) have methods when it is not possible to identify the distribution
- Classes of distributions may be an approach to make RBDOs more robust